

# Contents

## Number

Non-calculator tricks	4
Written multiplication & division	5
Special number sequences	6
Prime numbers	7
Multiples, factors & prime factorisation	8
LCM & HCF	9
Fractions	10
Fractions, decimals & percentages	12
Terminating & recurring decimals	14
Negative numbers	15
Powers & roots	16
Rational, irrational & surds	18
BIDMAS & bracket buttons	20
Percentages	21
Ratio & proportion	24
Rounding & estimating	26
Standard index form	28

## Algebra

Using letters	31
Simplifying expressions	31
Factorising expressions	32
Formulae	33
Rearranging formulae	34
Solving equations	35
Trial & improvement	36
Coordinates	37
Straight-line graphs	38
Inequalities	42
Equation of a circle	44
Simultaneous equations	45
Quadratics	49
Graphs you should know	54
Graph transformations	56
Real-life graphs	58
Direct & inverse proportion	60
Sequences	61

## Shape, space & measures

Symmetry & properties of shapes	62
Angles & parallel lines	64
Polygons	65
Areas of triangles & quadrilaterals	66
Circles	67
Composite shapes	68
Volume & surface area	69
Dimensions	73
Pythagoras' theorem	74
Trigonometry	76
Sine & cosine rules	79
3-D Pythagoras & trigonometry	81
Bearings	82
Converting between measures	83
Rounding measures	84
Calculating with rounded measures	85
Compound measures	86
Circle theorems	87
Constructions & loci	90
Congruent & similar shapes	92
Vectors	94
Transformations	96

## Handling data

Mean, median, mode, range	100
Two-way tables; stem & leaf diagrams	101
Pie charts	102
Time series & moving averages	103
Averages from frequency tables	104
Frequency diagrams	106
Cumulative frequency	108
Box plots	110
Comparing sets of data	111
Scatter graphs	112
Probability	113
Surveys & sampling	116
Speedy revision test	117
Answers	119
Index	126

## Equation of a circle

The equation of a circle **centred on the origin** with **radius  $r$**  is:

$$x^2 + y^2 = r^2$$

### ● Drawing a circle from its equation

#### ➤ Q & A

Draw the graph of  $2y^2 = -2x^2 + 8$ .

If there's an  $x^2$  and a  $y^2$  it's probably a circle.

#### Answer

##### ➤ Method

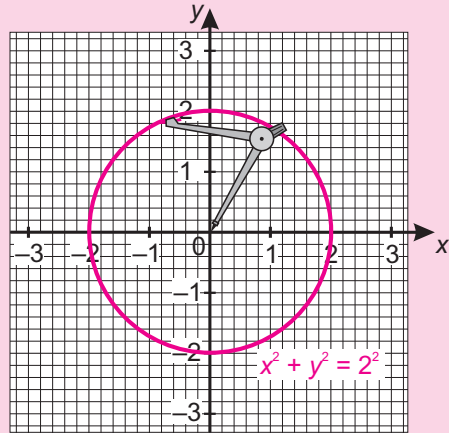
- Write the equation in the form  $x^2 + y^2 = r^2$ .
- Use your **compasses** to draw a circle **centred on the origin** with **radius  $r$** .

$$2y^2 + 2x^2 = 8 \quad [+2x^2]$$

$$y^2 + x^2 = 4 \quad [\div 2]$$

$$x^2 + y^2 = 2^2$$

So the radius,  $r$ , is 2 units.



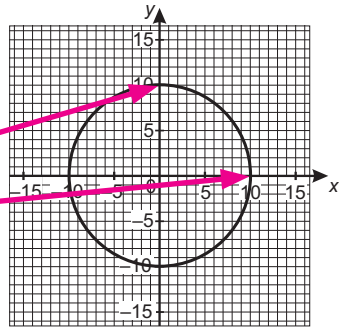
### ● Finding the equation of a circle from its graph

Just write down where the circle **cuts the positive x- or y-axis**.

This is the **radius  $r$** .

Here  $r$  is 10.

So the equation is  $x^2 + y^2 = 10^2$   
or  $x^2 + y^2 = 100$



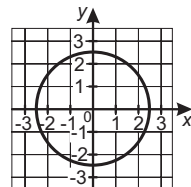
1 Draw the graphs of these equations:

a  $y^2 + x^2 = 400$

b  $y^2 = -x^2 + 81$

c  $x^2 + 2x + 4 = -y^2 + 2x + 20$

2 What is the equation of this circle?



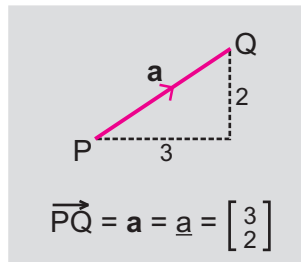
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# Vectors (1)

## ● Vector notation

A vector can be written in different ways:

- ◆ with an **arrow** showing where the vector goes **from** (P) and **to** (Q)
- ◆ using a **bold or underlined letter**
- ◆ in a **column** showing **distance right** (3) and **distance up** (2).



Note:  $-\mathbf{a}$  is the same length as  $\mathbf{a}$  but it goes in the opposite direction, i.e. from Q to P.

## ● Adding & subtracting column vectors

### ► Q & A

$$\mathbf{u} = \begin{bmatrix} -6 \\ 3 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

Find  $\mathbf{u} + \mathbf{v}$  and  $\mathbf{u} - \mathbf{v}$ .

Answer

$$\mathbf{u} + \mathbf{v} = \begin{bmatrix} -6 \\ 3 \end{bmatrix} + \begin{bmatrix} 5 \\ 8 \end{bmatrix} = \begin{bmatrix} -6 + 5 \\ 3 + 8 \end{bmatrix} = \begin{bmatrix} -1 \\ 11 \end{bmatrix}$$

$$\mathbf{u} - \mathbf{v} = \begin{bmatrix} -6 \\ 3 \end{bmatrix} - \begin{bmatrix} 5 \\ 8 \end{bmatrix} = \begin{bmatrix} -6 - 5 \\ 3 - 8 \end{bmatrix} = \begin{bmatrix} -11 \\ -5 \end{bmatrix}$$

### ► Method

- ① Add/subtract the **top numbers**.
- ② Add/subtract the **bottom numbers**.

The **sum of two vectors** is sometimes called the '**resultant vector**'.

## ● Multiplying column vectors by a 'scalar'

You can't multiply vectors together, but you can multiply a vector by a 'scalar' (i.e. any number).

### ► Q & A

Find  $3\mathbf{p}$ , where  $\mathbf{p} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ .

Answer

$$3\mathbf{p} = 3 \times \begin{bmatrix} 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \times 4 \\ 3 \times 5 \end{bmatrix} = \begin{bmatrix} 12 \\ 15 \end{bmatrix}$$

$3\mathbf{p}$  is **parallel** to  $\mathbf{p}$  and **3 times as long**.

### ► Method

Multiply top and bottom by the number.

- 1 Using  $\mathbf{u}$ ,  $\mathbf{v}$  and  $\mathbf{p}$  above, find as column vectors  $-\mathbf{u}$ ,  $3\mathbf{p} - 2\mathbf{v}$ .
- 2 What do you notice about your answer to 1?

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